1. INTRODUCTION

Quantum-state transfer between two distant parties is an important and rewarding task in quantum information processing and quantum communications. Several proposals have been put forward employing schemes based on cavity quantum electrodynamics (QED) [1–9]. More recently, quantum-state transfer in quantum optomechanics, where mechanical modes are coupled to the optical modes via radiation pressure, has become a subject of interest [4–9]. In particular, entanglement transfer between two spatially separated cavities is appearing in quantum information processing. Entangling two movable mirrors of an optical ring cavity [10], entangling two mirrors of two different cavities illuminated by entangled light beams [11] and entangling two mirrors of a double-cavity setup coupled to two independent squeezed vacua [12] have been considered. Recently, entangling two mirrors of a ring cavity fed by two independent squeezed vacua has been proposed [13]. This, however, cannot be used to implement long-distance entanglement transfer because the two movable mirrors belong to the same cavity.

In this work, we propose a simple model to entangle the states of two movable mirrors of spatially separated nanoresonators coupled to a common two-mode squeezed vacuum. The two-mode squeezed light, which can be generated by spontaneous parametric downconversion, is injected into the nanoresonators as biased noise fluctuations with non-classical correlations. The nanoresonators are also driven by two independent coherent lasers (see Fig. 1). The modes of the movable mirrors are coupled to their respective optical modes and to their local environments. Our analysis goes beyond the adiabatic regime [11] by considering the more general case of nonadiabatic regime and asymmetries between the laser drives as well as mechanical frequencies of the movable mirrors. Using parameters from a recent optomechanics experiment [14], we show that the states of the two initially independent movable mirrors can be entangled in the steady state as a result of entanglement transfer from the two-mode squeezed light. More interestingly, the entanglement in the two-mode light can be totally transferred to the relative position and the total momentum of the two movable mirrors when the following conditions are met: (1) the nanoresonators are resonant with the mechanical modes, (2) the resonator field adiabatically follows the motion of the mirrors, and (3) the optomechanical coupling is sufficiently strong. We also show that the entanglement transfer is possible in the nonadiabatic regime (low mechanical quality factor), which is closer to experimental reality. Unlike previous proposals [12,13], where a double or a ring cavity is considered, our scheme can be used, in principle, for a practical test of entanglement between two distant movable mirrors, for example, by connecting the squeezed source to the nanoresonators by an optical fiber cable. Given the recent successful experimental realization of strong optomechanical coupling [14] and availability of strong squeezing up to 10 dB [15], our proposal of efficient light-to-matter entanglement transfer may be realized experimentally.

2. MODEL

We consider two nanoresonators, each having a movable mirror and coupled to a common two-mode squeezed vacuum reservoir, for example, from the output of the parametric downconverter. One mode of the output of the squeezed vacuum is sent to the first nanoresonator and the other mode to the second nanoresonator. The movable mirror $M_j$ oscillates at frequency $\omega_{M_j}$ and interacts with the $j$th optical mode. The $j$th nanoresonator is also pumped by an external coherent drive of amplitude $\epsilon_j = \sqrt{2}\kappa_j P_j/\hbar\omega_{M_j}$, where $\kappa_j$ is the $j$th
nanoresonator damping rate, $P_j$ the drive pump power of the $j$th laser, and $\omega_{b_j}$ is its frequency. The schematic of our model system is depicted in Fig. 1. The system Hamiltonian has the form ($\hbar = 1$)

$$H = \sum_{j=1}^{2} \left[ \omega_{M_j} b_j^\dagger b_j + \omega_{l_j} a_j^\dagger a_j + g_j a_j^\dagger (b_j + b_j^\dagger) + (a_j^\dagger e^{i\phi_j} e^{-i\omega_{b_j} t} + a_j e^{i\phi_j} e^{i\omega_{b_j} t}) \right],$$

(1)

where $\omega_{l_f}$ is the $f$th movable mirror frequency, $q_f$ is the phase of the $f$th input field, and $g_f = (\omega_{l_f}/L_f) / \sqrt{\hbar/M_f \omega_{b_f}}$ is the single photon optomechanical coupling, which describes the coupling of the mechanical mode with the intensity of the optical mode [16], where $L_f$ is the length of the $f$th nanoresonator and $M_j$ is the mass of the $j$th movable mirror; $\omega_{b_j}$ is the frequency of the $j$th coherent pump laser; and $b_j$ and $a_j$ are the annihilation operator for the $j$th mechanical mode while $a_j^\dagger$ is the annihilation operator for the $j$th optical mode. Using the Hamiltonian (1), the nonlinear quantum Langevin equations for the optical and mechanical mode variables read [4,5,12]

$$\dot{b}_j = - \left( i\omega_{M_j} + \frac{\gamma_f}{2} \right) b_j - ig_j a_j^\dagger a_j + \sqrt{\gamma_f} f_j,$$

(2)

$$\dot{a}_j = - \left( i\omega_{l_j} - \frac{\gamma_j}{2} - i\Delta_j \right) a_j - ig_j a_j(b_j^\dagger + b_j) - i\epsilon_j e^{i\phi_j} + \sqrt{\kappa_j} F_j,$$

(3)

where $\gamma_j$ is the $j$th movable mirror damping rate, $\Delta_j = \omega_{b_j} - \omega_{l_j}$ is the laser detuning, and $f_j$ is a noise operator describing the coupling of the $j$th movable mirror with its own environment while $F_j$ is the squeezed vacuum noise operator. Note that Eq. (3) is written in a frame rotating with $\omega_{l_j}$. We assume that the mechanical baths are Markovian and have the following nonzero correlation properties between their noise operators [17,18]:

$$\langle f_j(\omega) f_j^\dagger(\omega') \rangle = 2\pi (n_{th,j} + 1) \delta(\omega + \omega'),$$

(4)

$$\langle f_j^\dagger(\omega) f_j(\omega') \rangle = 2\pi n_{th,j} \delta(\omega + \omega'),$$

(5)

where the movable mirrors are damped by the thermal baths of mean number of phonons $n_{th,j} = [\exp(h\omega_{b_j}/k_B T_j) - 1]^{-1}$.

The squeezed vacuum operators $F_j$ and $F_j^\dagger$ have the following nonvanishing correlation properties [13]:

$$\langle F_j(\omega) F_j^\dagger(\omega') \rangle = 2\pi (N + 1) \delta(\omega + \omega').$$

(6)

$$\langle F_j^\dagger(\omega) F_j(\omega') \rangle = 2\pi N \delta(\omega + \omega').$$

(7)

$$\langle F_1(\omega) F_2^\dagger(\omega') \rangle = 2\pi M \delta(\omega + \omega' - \omega_{M_1} - \omega_{M_2}).$$

(8)

$$\langle F_1^\dagger(\omega) F_2(\omega') \rangle = 2\pi M \delta(\omega + \omega' - \omega_{M_1} - \omega_{M_2}).$$

(9)

where $N = \sinh^2 r$ and $M = \sinh r \cosh r$ with $r$ being the squeeze parameter for the squeezed vacuum light.

3. LINEARIZATION OF QUANTUM LANGEVIN EQUATIONS

The coupled nonlinear quantum Langevin equations [Eqs. (2) and (3)] are in general not solvable analytically. To obtain an analytical solution to these equations, we adopt the following linearization scheme [17]. We decompose the mode operators as a sum of the steady-state average and a fluctuation quantum operator as $a_j = a_j + \delta a_j$ and $b_j = b_j + \delta b_j$, where $\delta a_j$ and $\delta b_j$ are operators. The mean values $a_j$ and $b_j$ are obtained by solving the expectation values of Eqs. (2) and (3) in the steady state:

$$a_j \equiv \langle a_j \rangle = -\frac{i\epsilon_j e^{i\phi_j}}{\kappa_j/2 - i\Delta_j^*},$$

(10)

$$b_j \equiv \langle b_j \rangle = -\frac{-i g_j |a_j|^2}{\gamma_j/2 + i\omega_{b_j}},$$

(11)

where $\Delta_j^* = \Delta_j - g_j(\beta_j + \beta_j^*)$ is the effective detuning, which includes the displacement of the mirrors due to the radiation pressure force. The contribution from the displacement of the movable mirrors is proportional to the intensity of the nanoresonator field, $n_j \equiv |a_j|^2$. In principle, we can arbitrarily choose the detunings $\Delta_j$ provided that we are away from the unstable regime [18].

Using $a_j = a_j + \delta a_j$ and $b_j = b_j + \delta b_j$, Eqs. (2) and (3) can be written as
\[ \delta b_j = \left( -i\omega_{\text{M}} + \frac{\gamma_j}{2} \right) \delta b_j + \mathcal{G}_j(\delta a_j - \delta a_j^\dagger) + \sqrt{\gamma_j} f_j, \]  
(12)

\[ \delta \hat{a}_j = -\left( \frac{\gamma_j}{2} - i\Delta_j \right) \delta a_j - \mathcal{G}_j(\delta b_j^\dagger + \delta b_j) + \sqrt{\kappa_j} \tilde{F}_j, \]  
(13)

where \( \mathcal{G}_j \equiv g_j|\alpha_j| = g_j \sqrt{\bar{n}_j} \) is the many-photon optomechanical coupling. Since the phase of the coherent drives can be arbitrary, for convenience we have chosen the phase of the input field to be \( \phi_j = -\arctan(2\Delta_j/\kappa_j) \) so that \( \alpha_j = -i|\alpha_j| \). Notice that the linearized Eqs. (12) and (13) can be described by an effective Hamiltonian \( (\hbar = 1) \)

\[ \mathcal{H} = \sum_{j=1}^{2} \left[ \omega_{\text{M},j} \delta b_j - \Delta_j^\prime \delta a_j \delta a_j + i\mathcal{G}_j(\delta a_j - \delta a_j^\dagger)(\delta b_j + \delta b_j^\dagger) \right] \]  
(14)

with a new effective many-photon optomechanical coupling \( \mathcal{G}_j \), which is stronger than the single photon coupling \( g_j \) by a factor of \( \sqrt{\bar{n}_j} \). The effective Hamiltonian (14) describes two different processes depending on the choice of the laser detuning \( \Delta_j^\prime \) [16]. Here we want to emphasize that \( \omega_{\text{M},j} \gg \Delta_j \) and \( \Delta_j \gg \kappa_j \) so that we can apply the rotating wave approximation. The latter is the case when the resonators are strongly off-resonant with the laser fields. When \( \Delta_j' = -\omega_{\text{M},j} \), within the rotating wave approximation, the interaction Hamiltonian reduces to \( \mathcal{H}_I = -i \sum_{j=1}^{2} \mathcal{G}_j(\delta a_j \delta b_j^\dagger - \delta a_j^\dagger \delta b_j) \), which is relevant for quantum-state transfer [4,5] and cooling (transferring of all thermal phonons into cold photon mode) [19]. In quantum optics, it is referred to as a beamsplitter interaction, whereas when \( \Delta_j' = +\omega_{\text{M},j} \) (in the rotating wave approximation), the interaction Hamiltonian takes a simple form \( \mathcal{H}_I = -i \sum_{j=1}^{2} \mathcal{G}_j(\delta a_j \delta b_j^\dagger - \delta a_j^\dagger \delta b_j) \), which describes parametric amplification interaction and can be used for efficient generation of optomechanical squeezing and entanglement. In this work, we are interested in quantum-state transfer and hence choose \( \Delta_j' = -\omega_{\text{M},j} \). Then, for \( \Delta_j' = -\omega_{\text{M},j} \) and in a frame rotating with frequency \( \omega_{\text{M},j} \) (neglecting the fast oscillating terms), one gets

\[ \dot{\delta b}_j = -\frac{\gamma_j}{2} \delta b_j + \mathcal{G}_j \delta a_j + \sqrt{\gamma_j} \tilde{F}_j, \]  
(15)

\[ \dot{\delta \hat{a}}_j = -\frac{\gamma_j}{2} \delta \hat{a}_j - \mathcal{G}_j \delta \hat{b}_j + \sqrt{\kappa_j} \tilde{F}_j, \]  
(16)

where we have introduced a notation for operators: \( \delta = o \exp(i\omega_{\text{M},j}t) \).

In the following section, we use these equations to analyze the entanglement of the states of the movable mirrors via entanglement transfer.

### 4. Entanglement Analysis

In order to investigate the entanglement between the states of the movable mirrors of the two spatially separated nanoresonators, we introduce two Einstein–Podolsky–Rosen (EPR)-type quadrature operators for the mirrors, namely their relative position \( X \) and the total momentum \( Y = X_1 - X_2 \) and \( Y = Y_1 + Y_2 \), where \( X_i = (\delta b_i + \delta b_i^\dagger)/\sqrt{2} \) and \( Y_i = i(\delta b_i^\dagger - \delta b_i)/\sqrt{2} \). We apply an entanglement criterion [20] for continuous variables, which is sufficient for non-Gaussian states and sufficient and necessary for Gaussian states. According to this criterion, the states of the movable mirrors are entangled if

\[ \Delta X^2 + \Delta Y^2 < 2. \]  
(17)

### A. Adiabatic Regime

An optimal quantum-state transfer (in this case from the two-mode squeezed vacuum to the mechanical motion of the mirrors) is achieved when the nanoresonator fields adiabatically follow the mirrors, \( \kappa_j \gg \gamma_j, \mathcal{G}_j \) [12], which is the case for mirrors with high-Q mechanical factor and weak effective optomechanical coupling. In fact the condition \( \kappa_j \gg \gamma_j \) can also be expressed as \( \omega_{\text{M},j} \gg \omega_{\text{M},j}(Q_{\text{th},j}/Q_{\text{M},j}) \) Inserting the steady state solution of Eq. (16) into Eq. (15), we obtain equations describing the dynamics of the movable mirrors:

\[ \dot{\delta \hat{b}}_j = -\frac{\gamma_j}{2} \delta \hat{b}_j + \sqrt{\Gamma_{a,j}} \tilde{F}_j + \sqrt{\gamma_j} \tilde{F}_j, \]  
(18)

where \( \Gamma_j = \Gamma_{a,j} + \gamma_j \) with \( \Gamma_{a,j} = 4\gamma_j^2/\kappa_j \) being the effective damping rate induced by the radiation pressure [21].

First, let us consider the variance of the relative position of the two mirrors \( \Delta X^2 \), which can be expressed as \( \Delta X^2 = \langle X^2 \rangle - \langle X \rangle^2 \). Since the noise operators corresponding to the two-mode squeezed vacuum \( F_j \) as well as the movable mirrors baths \( f_j \) have zero mean values, it is easy to show that \( \langle X \rangle = 0 \). Therefore, \( \Delta X^2 = \langle X_1^2 \rangle + \langle X_2^2 \rangle - 2 \langle X_1 \rangle \langle X_2 \rangle \). To evaluate these correlations, it is more convenient to work in the frequency domain. To this end, the Fourier transform of Eq. (18) yields

\[ \delta \hat{b}_j(\omega) = \frac{\sqrt{\Gamma_{a,j}} \tilde{F}_j(\omega) + \sqrt{\gamma_j} \tilde{f}_j(\omega)}{\Gamma_j/2 + i\omega}. \]  
(19)

The expectation value of the position \( X_1 \) of the first movable mirror can be expressed as

\[ \langle X_1^2 \rangle = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\omega d\omega' \exp(i\omega - i\omega') \langle X_1(\omega) X_1(\omega') \rangle. \]  
(20)

Using the correlation properties of the noise operators [Eqs. (2)–(5)], we obtain

\[ \langle X_1^2 \rangle = \frac{1}{2} \left( 2N + 1 \right) \frac{\Gamma_{a,1}}{\Gamma_1} + \frac{\gamma_1}{2\Gamma_1} \left( 2n_{\text{th},1} + 1 \right). \]  
(21)

Similarly, it is easy to show that

\[ \langle X_2^2 \rangle = \frac{1}{2} \left( 2N + 1 \right) \frac{\Gamma_{a,2}}{\Gamma_2} + \frac{\gamma_2}{2\Gamma_2} \left( 2n_{\text{th},2} + 1 \right). \]  
(22)

\[ \langle X_1 X_2 \rangle = \langle X_2 X_1 \rangle = \frac{2\sqrt{\Gamma_{a,1} \Gamma_{a,2}}}{\Gamma_1 + \Gamma_2} M. \]  
(23)

Therefore, using Eqs. (21)–(23), the variance of the relative position of the movable mirrors becomes
\[ \Delta X^2 = \frac{1}{2} (2N + 1) \left( \frac{\Gamma_n}{\Gamma_1} + \frac{\Gamma_a}{\Gamma_2} \right) - \frac{4\sqrt{\Gamma_n \Gamma_a}}{\Gamma_1 + \Gamma_2} M \]
\[ + \frac{\gamma_1}{2\Gamma_1} (2n_{th,1} + 1) + \frac{\gamma_2}{2\Gamma_2} (2n_{th,2} + 1). \] (24)

It is easy to show that the variance of the total momentum of the movable mirrors is the same as that of \( X \), i.e., \( \Delta X^2 = \Delta Y^2 \). Thus, the sum of the variances of the relative position and total momentum of the movable mirrors is given by
\[ \Delta X^2 + \Delta Y^2 = \frac{\gamma_1}{\Gamma_1} (2n_{th,1} + 1) + \frac{\gamma_2}{\Gamma_2} (2n_{th,2} + 1) \]
\[ + (2N + 1) \left( \frac{\Gamma_n}{\Gamma_1} + \frac{\Gamma_a}{\Gamma_2} \right) - \frac{8\sqrt{\Gamma_n \Gamma_a}}{\Gamma_1 + \Gamma_2} M. \] (25)

1. Identical Nanoresonators
To elucidate the physics of light-to-matter entanglement transfer, we first consider a simplified case of identical nanoresonators coupled to a two-mode squeezed vacuum. We also assume the external laser drives to have the same strength and the thermal baths of the two movable mirrors to be at the same temperature \( n_{th,1} = n_{th,2} = n_{th} \). To this end, setting \( \Gamma_1 = \Gamma_2 = \Gamma \), \( \Gamma_n = \Gamma_a = \Gamma_a \), \( M_1 = M_2 = M \), \( \omega_r = \omega_r = \omega_r \), \( \omega_M = \omega_M = \omega_M \), \( \kappa = \kappa = \kappa \), and \( \gamma_1 = \gamma_2 = \gamma \) and using the relation \( N = \sinh^2 r \), \( M = \sinh r \), the variance of the relative position \( (25) \) takes a simple form:
\[ \Delta X^2 + \Delta Y^2 = \frac{2\Gamma_a}{\gamma + \Gamma_a} e^{-2r} + \frac{2\gamma}{\gamma + \Gamma_a} (2n_{th} + 1) \]
\[ = \frac{8e^{-2r} \Gamma^2 / \gamma + 2 + 4n_{th}}{4\Gamma^2 / \gamma + 1} \]
\[ = \frac{2C}{C + 1} e^{-2r} + \frac{2(1 + 2n_{th})}{C + 1}, \] (26)
where \( C = 4\Gamma^2 / \gamma = 4\hbar \gamma^2 / \hbar \pi \) is the optomechanical cooperativity \([22]\). In the absence of the two-mode squeezed vacuum reservoir \( r = 0 \), Eq. \( (26) \) reduces to \( \Delta X^2 + \Delta Y^2 = 2 + 4n_{th} / (C + 1) \), which is always greater than 2, indicating the mechanical motion of the two mirrors cannot be entangled without the squeezed vacuum. This is because the motion of the mirrors is initially uncorrelated and their interaction via vacuum does not create correlations. In the limit \( C \gg 1 \) (a weaker condition \([22]\) for strong coupling regime), the sum of the variances can be approximated by \( \Delta X^2 + \Delta Y^2 \approx 2 \exp(-2r) + 4n_{th} / C \). Therefore, \( 4n_{th} / C < 1 \), which can be achieved for a sufficiently large number of photons in the nanoresonator, the sum of the variances can be less than 2 when
\[ r > \frac{1}{2} \ln[1/(1-2n_{th} / C)], \] (27)
indicating transfer of the quantum fluctuations of the input fields to the motion of the movable mirrors. This can be interpreted as entanglement transfer from light to mechanical motion. The interesting aspect is that this quantum-state-transfer scheme can, in principle, be extended to long-distance state transfer if the two nanoresonators are kept far apart but connected by, for example, an optical fiber cable to the output of the two-mode squeezed vacuum. Obviously, the entanglement between the mirrors would degrade when the distance between the resonators is increased owing to the decrease in degree of squeezing as a result of environmental couplings. Recently, similar transfer scheme from light to matter has been proposed \([1,24,25]\).

For realistic estimation of the entanglement between the movable mirrors, we use parameters from recent experiment \([14]\): laser frequency \( \omega_L = 2\pi \times 2.82 \times 10^{14} \text{Hz} (\lambda = 1064 \text{nm}) \), \( \omega_r = 2\pi \times 5.64 \times 10^{14} \text{Hz} (\omega_c = 2\omega_r) \), \( M_1 = M_2 = 145 \text{nm} \), \( L = 25 \text{mm} \), \( \kappa = 2\pi \times 215 \times 10^3 \text{Hz} \), \( \gamma = 2\pi \times 140 \text{Hz} \), and \( \alpha_M = 2\pi \times 947 \times 10^3 \text{Hz} \). In Fig. 2, we plot the sum of the variances of \( X \) and \( Y \) as a function of the thermal bath temperature of the movable mirrors. This figure shows that the movable mirrors are entangled when the nanoresonators are fed by squeezed light. Notice that based on the definition of the quadrature operators \( X \) and \( Y \), an optomechanical quadrature squeezing \([18,22,26]\) is achieved when \( \Delta X^2 < 1 \) or \( \Delta Y^2 < 1 \). This implies that whenever there is optomechanical squeezing, the two movable mirrors are always entangled. This shows a direct relationship between optomechanical squeezing and entanglement of the mechanical modes of the movable mirrors.

It is also interesting to see the dependence of the mirror-mirror entanglement on the pump laser power strength. Figure 3 shows that for a given squeeze parameter \( r \) and thermal bath temperature \( T \) of the movable mirrors, there exists a minimum pump power strength for which the movable mirrors are entangled. The minimum power required to observe mirror-mirror entanglement can be derived from Eq. \( (26) \) by imposing the condition that \( \Delta X^2 + \Delta Y^2 < 2 \), which yields
\[ C > \frac{2n_{th}}{1 - \exp(-2r)}. \] (28)
Using the explicit form of \( G \) in \( C = 4G^2 / \gamma \), we then obtain \( (r \neq 0) \)
\[ P > \frac{\alpha}{(1 - e^{-2r})(\exp[h\omega_M/k_B T] - 1)}. \] (29)
where \( \alpha \equiv \gamma_0 M_1 L^2 \omega_M ([\kappa/2]^2 + \omega_0^2) / 2\omega_r^2 \) is a factor that can be fixed at the beginning of the experiment (note here that

Fig. 2. Plots of the sum of variances \( \Delta X^2 + \Delta Y^2 \) versus bath temperature \( T \) of the movable mirrors for drive laser power \( P = 10 \text{ mW} \) and frequency \( \omega_L = 2\pi \times 2.82 \times 10^{14} \text{Hz} (\lambda = 1064 \text{nm}) \), mass of the movable mirrors \( M_1 = M_2 = 145 \text{ ng} \), frequency of the nanoresonator \( \omega_r = 2\pi \times 5.26 \times 10^{14} \text{Hz} \), length of the cavity \( L = 125 \text{ mm} \), the mechanical motion damping rate \( \gamma = 2\pi \times 140 \text{ Hz} \), mechanical frequency \( \kappa = 2\pi \times 215 \times 10^3 \text{Hz} \), nanoresonator damping rate \( \gamma = 2\pi \times 947 \times 10^3 \text{Hz} \), and for different values of the squeezing parameter \( r \): 0.5 (blue solid curve), 1.0 (red dashed curve), and 2.0 (green dotted curve). The blue dashed line represents \( \Delta X^2 + \Delta Y^2 = 2 \).
$M_1 = M_2$). It is easy to see from Eq. (29) that for a given thermal bath temperature $T$ of the movable mirrors, increasing $r$ decreases the minimum power required to achieve entanglement between the mirrors.

When the number of thermal bath phonons increases, the minimum value of the cooperativity parameter for which the entanglement occurs increases. In the weak coupling regime, where the optomechanical cooperatively is much less than one, $C \ll 1$, the sum of the variances (26) that characterize the entanglement can be approximated by $\Delta X^2 + \Delta Y^2 \approx 2 + 2Cr^2 + 4n_{th}$. This is always greater than 2, independent of the degree of squeezing of the input field, indicating no quantum-state transfer from the squeezed light to the mechanical motion of the movable mirrors, and hence the mirrors remain unentangled. Figure 4 shows the plot of the entanglement measure versus the optomechanical cooperativity as a function of the thermal bath phonon numbers. For $r = 1.0$ and $n = 1.0(62.2 \mu K)$, the motion of the two mirrors are not entangled up to $C = 2n_{th}[1-\exp(-2r)]^{-1} \approx 2.3$.

2. Effect of Asymmetric Coherent Drives and Mechanical Frequencies

We next analyze the effect of the asymmetries in the strength of coherent drives and in the vibrational frequencies of the movable mirrors. Figure 5(a) illustrates that for a constant thermal bath temperature $T_1 = T_2 = 0.25 \text{ mK}$ of the movable mirrors and the squeeze parameter $r = 2.0$, there exist input laser powers $P_1$ and $P_2$, where $\Delta X^2 + \Delta Y^2$ is minimum or the entanglement is the strongest. It turns out that for identical nanoresonators, strong entanglement is achieved when $P_1 = P_2$. Notice also that the width of the entanglement region is mainly determined by the input power. The higher the input powers, the wider the width of the entanglement region becomes. Figure 5(b) shows optimized $\Delta X^2 + \Delta Y^2$ versus the input power $P_2$ for a given $P_1$ and for different values of the thermal bath temperatures $T_1$ and $T_2$. As expected the entanglement degrades as the thermal bath temperatures of the mirrors increase, and the entanglement persists at higher temperatures for sufficiently strong pump power strength (see green-dotted curve for $T_1 = T_2 = 0.5 \text{ mK}$).

Tuning the frequencies of the movable mirrors also affects the degree of the mirror–mirror entanglement. As shown in Fig. 6(a), for fixed temperatures of the thermal bath of the movable mirrors $T_1 = T_2 = 0.25 \text{ mK}$, squeeze parameter $r = 2.0$, drive powers $P_1 = P_2 = 11 \text{ mW}$, and frequency $\omega_{M_1}$ of the first movable mirror, there exists a frequency $\omega_{M_2}$ of the second movable mirror for which the entanglement is maximum. The smaller $\omega_{M_1}$ is, the stronger the entanglement becomes. The optimum entanglement decreases with increasing frequency $\omega_{M_1}$ of the first movable mirror and eventually disappears at sufficiently large $\omega_{M_1}$ and relatively high temperatures [see Fig. 6(b)].

B. Nonadiabatic Regime

So far we have discussed the mirror–mirror entanglement induced by the squeezed light in the adiabatic regime ($\kappa_j \gg \gamma_j, \phi_j$). We next derive a condition for entanglement valid for both adiabatic and nonadiabatic regimes. We also study the field–field entanglement in the regime where the two mirrors are entangled.
The dynamics of the movable mirrors in the nonadiabatic regime is described by the coupled Eqs. (15) and (16). Solving the Fourier transforms of these equations yields

$$\delta \hat{b}_J = \frac{\kappa J_f}{d_j(\omega)} \sqrt{\gamma J_f} \hat{J}_f + \frac{G_j}{d_j(\omega)} \sqrt{\kappa} \hat{P}_j,$$

where $d_j(\omega) = G_j^2 + (\gamma J_f/2 + i\omega)(\kappa J_f/2 + i\omega)$. Thus using Eq. (30) and the properties of the noise operators (4)-(9), the sum of the variances of the relative position $X$ and total momentum $Y$ of the movable mirrors (for identical nanoresonators) is found to be

$$\Delta X^2 + \Delta Y^2 = \frac{2C}{C + 1} \frac{\kappa e^{-2\gamma}}{\kappa + \gamma} + \frac{2(2n_{th} + 1)}{C + 1} \left[ 1 + \frac{C\gamma}{\kappa + \gamma} \right].$$

We immediately see that for $\kappa \gg \gamma, C$, Eq. (31) reduces to the expression (26) derived in the adiabatic approximation. In general, for the dissipation rate of the movable mirrors $\gamma J_f$ comparable to the resonator decay $\kappa J_f$, the expression (31) can be significantly different from (26).

In Fig. 7 we present a comparison showing the entanglement transfer in the adiabatic and nonadiabatic regimes. The main difference comes from the mechanical dissipation rate $\gamma$. Since the adiabatic approximation assumes negligible mechanical dissipation rate, the transfer is more efficient than the nonadiabatic case. This, however, is an ideal situation, which requires a very high mechanical quality factor. In general, for a low mechanical quality factor the mechanical dissipation can be significant, leading to a less efficient entanglement transfer. As can be noted from Fig. 7, the mirror-mirror entanglement diminishes when the normalized mechanical dissipation rate $\gamma/k$ increases from 0.01 to 0.05. We note that when the dissipation rate increases, a large cooperativity (strong coupling) is required to observe the mirror–mirror entanglement.

To gain insight into the transfer of entanglement from the squeezed light to the motion of the mirrors, it is important to study the entanglement between the optical modes of the nanoresonators. This can be analyzed by introducing two EPR-type quadrature operators $x = x_1 - x_2$ and $y = y_1 - y_2$, where $x_1 = (\delta \hat{a}_1 + \delta \hat{a}_1^\dagger) / \sqrt{2}$ and $y_1 = i(\delta \hat{a}_1^\dagger - \delta \hat{a}_1) / \sqrt{2}$. The optical modes of the nanoresonators are entangled if

$$\Delta x^2 + \Delta y^2 < 2.$$  

Solving the Fourier transforms of Eqs. (2) and (3), we obtain

$$\delta \hat{a}_j(\omega) = -\frac{G_j}{d_j(\omega)} \sqrt{\gamma J_f(\omega)} + \gamma J_f/2 + i\omega \frac{G_j}{d_j(\omega)} \sqrt{\kappa} \hat{P}_j,$$

where $d_j(\omega) = G_j^2 + (\gamma J_f/2 + i\omega)(\kappa J_f/2 + i\omega)$. The sum of the variances of $x$ and $y$ for identical nanoresonators reads

$$\Delta x^2 + \Delta y^2 = \frac{2C(2n_{th} + 1)}{C + 1} \frac{\gamma}{\gamma + \kappa} + 2 \left( \frac{k}{k + \gamma} + \frac{1}{C + \gamma} \right) e^{-2\gamma}.$$  

which for the case $\gamma/k \ll 1$ and strong coupling regime ($C \gg 1$) reduces to

$$\Delta x^2 + \Delta y^2 \approx 2(2n_{th} + 1) \frac{\gamma}{\gamma + k} + 2e^{-2\gamma}.$$  

We note from Eq. (35) that in the strong coupling regime, the field–field entanglement is mainly determined by the thermal bath temperature and the squeeze parameter, not on the value of $C$. For experimental parameter in Ref. [10] we have $\gamma/k = 6.5 \times 10^{-4}$, and assuming the thermal bath mean phonon number $n_{th} = 5$, the field–field entanglement is insensitive to the increase of the cooperativity while the entanglement of the states of the movable mirrors increases as the optomechanical coupling becomes stronger or when $C$ increases (Fig. 8). It is interesting to see that for a sufficiently strong coupling (large values of cooperativity, $C$), the entanglement between the...
states of the movable mirrors can be as strong as that of the squeezed light. Therefore, in addition to choosing the mechanical frequency to be $\Delta' = -\omega_m$ and adiabatic approximation ($\kappa \gg \gamma, \tilde{g}$), it is imperative to attain strong coupling regime to achieve the maximum entanglement between the states of the movable mirrors.

Experimentally, the entanglement between the states of the movable mirrors can be measured by monitoring the phase and amplitude [10] of the transmitted field via the method of homodyne detection, in which the signal is brought into interference with a local oscillator that serves as phase reference. For other variants of optical measurement schemes, see Ref. [16]. With the availability [15] of strong squeezing sources up to 10 dB squeezing (90%) below the standard quantum limit, our proposal may be realized experimentally.

5. CONCLUSION

In summary, we have analyzed a scheme to entangle the vibrational modes of two independent movable mirrors of two spatially separated nanoresonators via two-mode squeezed light. We showed that in the regime of strong coupling $C \gg (4g^2 \gg \kappa \gamma)$ and when the nanoresonator field adiabatically follows the motion of the mirrors, the quantum fluctuations of the two-mode squeezed light is transferred to the motion of the movable mirrors, creating stationary entanglement between the vibrational modes of the movable mirrors. It turns out that an entanglement of the states of the movable mirrors as strong as the entanglement of the two-mode squeezed light can be achieved for a sufficiently large optomechanical cooperativity $C$ or equivalently for a sufficiently strong optomechanical coupling. We also considered a less stringent condition—nonadiabatic regime that is more realistic than the adiabatic approximation and still obtained entanglement transfer from the two-mode light to the movable mirrors. Given the recent successful experimental realization of strong optomechanical coupling [14] and well-developed method of homodyne measurement, our proposal for an efficient light-to-matter entanglement transfer may be realized experimentally.

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